Bio-inspired Time-to-contact Control for Autonomous Quadrotor Vehicles

Benjamin Thomsen*, Mingfeng Zhang† and Inna Sharf‡

McGill University, Montreal, Quebec H3A 2K6, Canada

This work develops and assesses a bio-inspired flight guidance and control system for quadrotor UAVs. It has been hypothesized that many animals use the perceived instantaneous time-to-contact with a target as the variable controlling certain time-sensitive maneuvers. This research applies time-to-contact control for the autonomous landing of quadrotor UAVs. Strategies are developed for the planning and control of two-dimensional landing maneuvers under a time-to-contact control framework. Through simulations and experimentation on a research quadrotor platform, time-to-contact control is implemented and assessed. This control methodology is determined to have benefits in its ability to arrive at a destination in a predefined time, but has certain practical limitations compared to conventional PID position control.

I. Introduction

The development of guidance, navigation, and control systems for unmanned aerial vehicles (UAVs) is motivated by the need for robust “autopilot” systems to facilitate reliable unmanned operation. Autonomous systems must be able to detect and react to disturbances and irregularities in real time, and must be functional in any environment the vehicle may encounter in use. As the field of unmanned aerial systems matures, many opportunities exist for exploration and experimentation in the autonomous control of UAVs.

Many purposes for which robots are developed intend to replicate the actions of animals. Replicating the neurological functions of animals in the controllers of autonomous robots can thus be an intuitive means to accomplish these purposes. Based on research in the fields of perception and movement control, it has been demonstrated that some animals, including humans, base the control of many of their point-to-point maneuvers on the time to contact with a spatial target. Rather than calculating position and velocity directly, the time-to-contact with a target is computed based on optical signals in the brain and then manipulated by internal control systems. This paper deals with the maneuvering of a quadrotor helicopter UAV through the manipulation of the instantaneous time-to-contact, or Tau, with a spatial target. We develop a framework for time-to-contact-based guidance and control of point-to-point maneuvers starting and ending at rest.

The following definitions for guidance, navigation, and control systems are used for the autonomous flight systems described in this research. The guidance system generates the desired state of the UAV, and is responsible for tasks such as trajectory generation, path planning, and decision-making, depending on the level of autonomy in the system. The navigation system collects and processes data from sensor inputs to estimate the vehicle’s state, and in certain cases its environment. The control system compares outputs from the guidance and navigation systems actuate the UAV and bring it closer to the desired state.

The motivation for the research described in this paper is to extend on the findings of published research with the aim of improving time-to-contact-based controller design, and to assess the extent to which a time-to-contact control scheme is practical. In this paper, we present a general point-to-point guidance and trajectory generation system based on the bio-inspired Tau theory. We analyze Tau control challenges to identify strategies for robust and stable controller design. Numerous novel control solutions to aid in timing control are developed and compared. These findings are integrated in a time-to-contact controller...
that is implemented and tested on the Draganfly X4P quadrotor platform at McGill University’s Aerospace Mechatronics Lab.

II. Literature review

This section will review the aforementioned topic introduced in the previous section (quadrotor vehicles, autonomous landing, control of landing maneuvers), and then narrow the scope to time-to-contact control. A more focused literature review of the biological origins of time-to-contact control, and then applications of time-to-contact control for quadrotors will follow.

II.A. Quadrotor control

- Quadrotor landing research in general
- Quadrotor control, specifically for landing maneuvers

II.B. Time-to-contact control

- Time-to-contact guidance and control in animals, its history, main research findings, scenarios in which it applies, etc
- Applications of time-to-contact control in mobile robotics, especially quadrotors and other aerial UAVs

III. Time-to-contact guidance for quadrotor landing

III.A. Intrinsic Tau guidance

Tau theory lays out the principles of movement guidance and control for the closure of a spatial distance, or gap, based on the time to contact with a destination in space. Research has shown that the closure of these gaps in some animals is controlled based on perceptual information about the gap and its instantaneous time-to-closure called Tau. If $x$ is the coordinate defining distance to the target, then the instantaneous time-to-closure of this gap is defined as:

$$\tau(t) = \frac{x(t)}{\dot{x}(t)}$$  \hspace{1cm} (1)

Tau theory posits that this is the variable that is controlled by neuronal circuits in animals, and there is thus a “reference” Tau guiding the closure of these motion gaps. This reference Tau varies with different types of maneuvers or gap closures, but for the purposes of this research we will consider the case of a maneuver starting and finishing at rest.

Mathematical relations have been developed based on observations of animal behavior that represent the reference Taus followed by animals for different maneuvers. It is hypothesized that for maneuvers starting and finishing at rest, the following function is generated in the nervous systems of animals for time-to-contact control:

$$\tau_g(t) = \frac{k}{2} \left( t - \frac{t_f^2}{t} \right)$$  \hspace{1cm} (2)

For $k = 1$, this function is analogous to the instantaneous time-to-contact with zero initial velocity and constant acceleration. Lee (1998) refers to this function for $\tau$ as “intrinsic Tau guidance”, $\tau_g$. In equation (2), $t_f$ represents the desired time to complete the maneuver, from start to finish, and $k$ is a constant of proportionality. From this reference function for instantaneous time-to-contact, corresponding equations for remaining gap distance, velocity, and acceleration can also be derived as follows:

$$x(t) = \frac{x_0}{t_f^{2/k}} \left( \frac{t_f^2}{k} - t^2 \right)^{1/k}$$  \hspace{1cm} (3)

$$\dot{x}(t) = \frac{-2x_0 t}{kt_f^{2/k}} \left( \frac{t_f^2}{k} - t^2 \right)^{1/k - 1}$$  \hspace{1cm} (4)

$$\ddot{x}(t) = \frac{-2x_0}{k^2t_f^{4/k}} \left( k t_f^2 - (2 - k)t^2 \right) \left( \frac{t_f^2}{k} - t^2 \right)^{1/k - 1}$$  \hspace{1cm} (5)
where $x_0$ denotes the gap distance at time $t = 0$. These reference functions are plotted in figure [1] with $t_f = 10$ s and $k = 0.3$. The position and velocity profiles as functions of time are then a sigmoid function and bell-shaped function, respectively. These functions are sufficient for the closure of a single gap (a one-dimensional maneuver), but more complex maneuvers involving multiple degrees of freedom generally require more than a single reference function.

![Figure 1: Reference functions for a general 1-D maneuver with intrinsic Tau guidance](image)

It can be seen in figure [1] and equation (2) that the reference Tau has infinite magnitude at $t = 0$. The simplest way to resolve this singularity is to replace $t$ with $t + t_\delta$ in equation (2) for small $t_\delta$ ($t_\delta << t_f$). This generally improves the control of the system at the start-up of the maneuver, but has negligible effect on the subsequent performance of the Tau guidance strategy.

### III.B. Two-dimensional Tau guidance

The movement of quadrotors is often more temporally or spatially efficient when controlled in multiple dimensions simultaneously. Any maneuver with a desired axis of approach to the destination, for example, will have at least two degrees of freedom. In the scheme of Tau control, *Tau coupling* is used to simultaneously close multiple gaps. Tau coupling is the process of defining the reference time-to-contact of one gap as being linearly proportional to the reference time-to-contact of another gap. The closure of a second gap, say in coordinate $y$, can be defined with the relation:

$$\tau_y(t) = k_{yx} \tau_x(t)$$

where $\tau_x$ is defined by the intrinsic Tau guidance of equation (2).

Many point-to-point maneuvers have a desired direction for the final approach to the target. For example, the desired behavior for quadrotor landing is generally to have small horizontal and angular velocities relative to the descent velocity. Zhang et al. (2014) propose the coupling of a distance gap and a pitch angle gap to achieve this type of behavior. For quadrotor landings with a vertical final approach, we propose the closure of an angular gap and a vertical gap, i.e., gap in the direction of the final approach, simultaneously, thus, differently from the coupling approach taken by Zhang et al. Figure 2 shows a sample transverse Tau landing. The angular gap, $\gamma$, is given by the angle between the desired axis of approach (the vertical axis) and the line between the current location of the quadrotor and the destination. By coupling the angular Tau and vertical Tau, this angle will go to zero at the same time as the vertical distance, achieving a vertical landing at the destination.

Starting with the Tau coupling, $\tau_\gamma = k_{\gamma z} \tau_z$, where $z$ is the vertical gap distance, and in a manner identical to how we derived the reference functions for a one-dimensional gap above, the reference functions for the
The direct coupling for two-dimensional maneuvers is thus between \( \tau_z \) and \( \tau_\gamma \); however, for controlling the quadrotor UAV, it is more convenient to map the angular reference function onto a coordinate normal to the vertical coordinate, i.e., \( x \perp z \). The guidance is thus in coordinates \( x \) and \( z \), which is more intuitive for quadrotor control. Corresponding reference functions for this gap (the horizontal gap in figure 2), are derived by using the relation \( x = z \tan (\gamma) \). The full forms of these functions are left out here for brevity, but are equivalent to the following relationships:

\[
x(t) = z(t) \tan (\gamma(t)) \tag{11}
\]
\[
\dot{x}(t) = \frac{z(t) \tan (\gamma(t)) + z(t) \dot{\gamma}(t) \sec^2 (\gamma(t))}{\dot{z}(t) \tan (\gamma(t)) + z(t) \dot{\gamma}(t) \sec^2 (\gamma(t)) + z(t) \dot{\gamma}(t) \sec^2 (\gamma(t)) + 2z(t) \dot{\gamma}(t) \tan (\gamma(t)) \sec^2 (\gamma(t))} \tag{12}
\]

The resulting equation for the horizontal Tau is then:

\[
\tau_x(t) = \frac{z(t) \tan (\gamma(t))}{\dot{z}(t) \tan (\gamma(t)) + z(t) \dot{\gamma}(t) \sec^2 (\gamma(t))} \tag{14}
\]

By coupling \( \tau_x \) and \( \tau_z \), as described above, we provide the flight controller with reference equations for two-dimensional point-to-point maneuvers with a specified direction of final approach to the destination. A quadrotor landing is a specific case of this where, usually, the desired direction of approach is aligned with the vertical axis. Alternate directions of final approach can also be considered for other applications of point-to-point Tau guidance.

One interesting application is for landing the quadrotor on angled surfaces. When flying quadrotors outdoors or in certain environments, a flat surface for landing is not always guaranteed. Mathematically, this time-to-contact guidance scheme is generated by subtracting the angle between the surface and horizontal from \( \gamma(t) \) of equation \( 8 \), and replacing \( \gamma_0 \) with the accordingly revised angle at time \( t = 0 \) in equations \( 8 \)
Figure 3: Reference functions for the Tau landing maneuver shown in figure 2 and (9) for use in (14). Again, these trajectories are derived from a specified finish time of the maneuver, allowing for maneuvers with high temporal precision.

The above reference equations for time-to-closure of motion gaps (both translational and angular) can be used in guidance systems to generate references for the control of planar maneuvers with a desired completion time and approach angle. The reference profiles for the Tau landing in figure 2 are plotted in figure 3. Larger accelerations and velocities are achieved in the horizontal direction, as this gap closes faster to allow for a vertical final approach to the destination. The control strategies that allow the quadrotor vehicle to follow the reference Tau functions for time-to-closure constitute a separate problem, discussed in the following section.

IV. Time-to-contact control for quadrotor landing

IV.A. Controller design framework

The quadrotor vehicles for which this controller is developed are controlled by throttle, which sends an equal signal to all four rotors, pitch, which sends a differential signal to the front/rear rotors, roll, which sends a differential signal to the left/right rotors, and yaw, which sends a differential signal to the clockwise/counterclockwise rotating rotors (see figure 4). The quadrotor is an underactuated system, as it has four actuators and six degrees of freedom. The attitude of the vehicle is completely controlled by the Euler angles (yaw, pitch, roll), but lateral and longitudinal translation are controlled indirectly with a combination of throttle and attitude control.

IV.A.1. Tandem-rotor model

As the scope of this research is limited to two-dimensional maneuvers, we will use a simplified three degree-of-freedom model of our target plant, which we call the tandem-rotor vehicle. The tandem-rotor in its simplest form is comprised of two parallel thrusters at either end of a plank of mass $m$ and moment of inertia $I$. The angle of the vehicle body with respect to the horizontal is the pitch angle, $\theta$. The thrusts from the left and right thrusters are $u_L$ and $u_R$, respectively and their differential thrust causes a moment, $M$, that rotates the vehicle. A diagram of the tandem-rotor model is shown in figure 5. This model allows for simulation of the translational and rotational dynamics of the quadrotor within a vertical plane, using the following
equations of motion:

\[
\begin{align*}
    m\ddot{x} &= (u_L + u_R) \sin \theta & (15) \\
    m\ddot{z} &= (u_L + u_R) \cos \theta - mg & (16) \\
    I\ddot{\theta} &= \frac{l}{2}(u_L - u_R) = M & (17) \\
\end{align*}
\]

where \( l \) is the distance between the two thrusters. As is the quadrotor vehicle, this model captures the underactuated non-holonomic nature of the system.

Using the equations of translational motion to eliminate the total thrust of the tandem-rotor, one can derive the relationship between the pitch of the vehicle and its acceleration components \( \ddot{x} \) and \( \ddot{z} \): indeed this gives the non-holonomic constraint on the vehicle’s motion. Then, by substituting for the acceleration components in terms of the analytical functions derived for the time-to-contact guidance system, an analytical solution for the desired pitch angle for a point-to-point maneuver, \( \theta_d \), can be derived as:

\[
\theta_d = \tan^{-1}\left( \frac{\ddot{x}}{\ddot{z} - g} \right) 
\]
Figure 6: Block diagram from tandem-rotor system

with $\dot{z}$ and $\ddot{z}$ defined as per results of section III.B. It is interesting to compare the above analytical desired pitch with the reference pitch angle calculated from the Tau controller outputs. The reference pitch angle from the Tau controller output ($\theta_{\text{ref}}$), the actual pitch angle from a simulation ($\theta$) and the analytical desired pitch angle ($\theta_d$) are plotted for a sample two-dimensional maneuver in figure 7. The parameters for the tandem-rotor model and Tau controllers are given in tables 1 and 2, respectively. Pitch angle control is one of the reasons that it is important to ensure that there is no chattering, or severe oscillations, in the Tau controller outputs. Chattering in these outputs would cause chattering in the reference pitch angle, which causes instabilities in the rotational dynamics of the vehicle.

Figure 7: Pitch simulation for a two-dimensional Tau control maneuver with the tandem-rotor model

IV.B. Tau controller error definitions

The function of Tau controller is to send signals to the quadrotor’s actuators to match the vehicle’s instantaneous time-to-contact to the reference time-to-contact generated by the guidance system, for each degree of freedom. Thus, the error value to be manipulated by the controller to determine its output is the instantaneous time-to-contact error. A standard definition of the error, used for example in a PID control law, is the arithmetic difference between the desired and actual values of the controlled variable. However, several aspects of Tau control make the usage of this conventional linear error problematic, as explained in the following.

The challenges of Tau control in general stem from three interrelated characteristics of time-to-contact “measurement”, specifically, the fact that in practice $\tau$ is computed by dividing the distance gap state by the gap rate state ($\tau = x/\dot{x}$). First, there is a singularity when the gap rate is zero at which point Tau becomes undefined. In practice, for a quadrotor vehicle, this means that Tau control is unstable while hovering or accelerating/decelerating from near zero velocities. This particular issue is aggravated by the fact that hover is generally the “idling” behavior of a quadrotor, but as noted, it is not a stable state for Tau control. Second, over a maneuver starting and ending at rest, the magnitude of the reference Tau will go from infinity to zero, which makes the use of a linear error definition difficult. A 10% difference between the measured and reference Taus at the beginning of the maneuver would be many orders of magnitude larger than a 10%
difference towards the end of the maneuver. Finally, a particular value of Tau does not correspond to a single state of the vehicle, as a number of different positions and velocities will have the same Tau value.

Motivated by the above considerations and the work of Kendoul (2014), we considered other nontraditional error definitions for Tau control. Kendoul (2014) developed controllers based on what are referred to as hybrid and non-linear ratio Tau control laws. In our research, we introduce two Tau control error definitions with similar goals to those of Kendoul, but different definitions. The following three error definitions for time-to-contact control are considered:

Standard difference error:

\[ e = \left( \tau - \tau_{ref} \right) \text{sign}(\dot{x}) \] (22)

Nonlinear error:

\[ e_N = \left( \frac{\tau - \tau_{ref}}{\tau_{ref}} \right) \text{sign}(\dot{x}) \] (23)

Hybrid-nonlinear error:

\[ e_{NH} = \begin{cases} \left( \frac{\tau - \tau_{ref}}{\tau_{ref}} \right) \text{sign}(\dot{x}) & \text{for } |\tau_{ref}| > \tau_{sw} \\ \left( \frac{\tau - \tau_{ref}}{|\tau_{ref}|} \right) \text{sign}(\dot{x}) & \text{for } |\tau_{ref}| \leq \tau_{sw} \end{cases} \] (24)

where \( \tau_{sw} \) introduced in the definition of the hybrid-nonlinear error is the switching point (to be defined by the user) between the two regimes to avoid discontinuity in the error signal. In these error definitions, \( \tau_{ref} \) refers to the Tau guidance function used as reference, and \( \tau \) refers to the measured value of Tau, which as noted earlier, is computed from the vehicle’s directly measured or observed states. The error definitions must be multiplied by the sign of the current velocity of the vehicle to ensure that controller output is of the proper sign. The nonlinear error definition above allows for the computation of the error on a relative rather than absolute basis. With this error definition, a 10% difference between reference and measured Taus will produce the same error at the beginning and end of the maneuver. The hybrid-nonlinear error definition provides the benefits of the nonlinear error \( e_N \) at the beginning of the maneuver, with the added stability as \( \dot{x} \to 0 \) towards the end of the maneuver.

IV.C. Implementation of Tau controller

Several aspects of the implementation of Tau controller are discussed in this section. For the results obtained with simulations of the tandem-rotor vehicle, the following parameters are used:

<table>
<thead>
<tr>
<th>Table 1: Tandem-rotor model parameters</th>
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<tbody>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>2.5 kg</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Tandem-rotor Tau controller parameters</th>
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<tbody>
<tr>
<td>( k_{p,z} )</td>
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<tr>
<td>( 50 \ \frac{\text{rad}}{\text{m}} )</td>
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IV.C.1. Tandem-rotor simulations

The tandem-rotor vehicle model has proven to be a convenient tool to investigate the effect of different control strategies on controller performance. One use of the tandem-rotor simulations is to compare Tau controller performance for the three errors defined in section IV.B. For the vertical landing maneuvers under \( \tau_z \)-controller, simulations showed a negligible difference in performance between the nonlinear and hybrid-nonlinear error definitions, but significant differences between the standard difference error \( e \) and the nonlinear error \( e_N \).

A comparison between the controller based on these two error definitions is presented in figure 8 for a vertical landing maneuver. The linear controller produces chattering in the output signal, which can be seen in figure 8. Saturating the controller output lessens this oscillatory behavior but does not completely remove it. This comparison demonstrates that the nonlinear error definition is more suitable for time-to-contact control, which is intuitive from a conceptual standpoint.
Two-dimensional landings can involve “hovering” near zero-velocity in one dimension, close to the singularity in Tau. The hybrid-nonlinear error definition in theory offers the benefits of the nonlinear error controller plus the greater stability near the singularity. Figure 8 shows a comparison between the controllers based on the nonlinear and hybrid-nonlinear error definitions. The main difference is that the nonlinear error results in chattering in the thrust signal as \( \dot{x} \to 0 \) whereas the hybrid-nonlinear error does not.

The attenuation in this chattering seen in the nonlinear error response is due to an increasingly strict saturation on the controller output that is necessary for the vehicle model to come to rest at the end of the maneuver. This saturation was only required for the controller using the nonlinear error, and not for the hybrid-nonlinear error. The controller using the hybrid-nonlinear error copes well with “hovering” at the end of the maneuver, removing the chattering from the thrust signal. The hybrid-nonlinear error definition thus demonstrates the best performance of the three error definitions considered for the time-to-contact control of point-to-point maneuvers.

The tandem-rotor model was also used for preliminary tuning of Tau controller PID gains. Tandem-rotor simulations showed that a gain on the derivative of the error, \( k_d \), degrades the performance of the controller. The derivative term was eliminated, and the resulting PI controller was tuned using the tandem-rotor model with parameters matching those of the physical vehicle used for flight tests, to be described in section V.

**IV.C.2. Gain scheduling**

Gain scheduling is one strategy to cope with plant nonlinearities by varying the control gains based on the value of a scheduling variable. There are multiple levels for applying gain scheduling within the design of Tau controllers. At the highest level, our Tau control scheme uses gain scheduling based on the parameters of the desired maneuver: the distance gap to be closed and the time to complete the gap closure. Using a one-dimensional vertical landing maneuver as an illustration, landing from 5m will require more aggressive control gains if it is to be completed in 5s rather than 10s, for example. To accomplish this, \( z_0 \) and \( t_f \) are
used as scheduling variables such that:

\[(k_p, k_i) \propto \frac{|z_0|}{t_f}\]  \hspace{1cm} (25)

that is, the control gains are scheduled in proportion to the average rate of gap closure. This gain scheduling can easily be extended to maneuvers with multiple degrees of freedom, by applying the relation above to the Tau controller of each gap to be closed.

At a lower level, we schedule control gains by dividing a maneuver into discrete blocks of time – normalized by the desired final time, \(t_f\) – and assigning separate gains to each block. This gain scheduling scheme is used to apply lower gains to the error at the beginning and end of the maneuver, when the vehicle is operating at low velocities, close to the singularity in Tau. In this gain scheduling scheme, \(t_{sw,1}\) and \(t_{sw,2}\) are switching points for the gains, to be determined based on the type of maneuver.

\[(k_p, k_i) = \begin{cases} (k_{p,1}, k_{i,1}) & \text{for } 0 \leq \frac{t}{t_f} \leq t_{sw,1} \\ (k_{p,2}, k_{i,2}) & \text{for } t_{sw,1} < \frac{t}{t_f} \leq t_{sw,2} \\ (k_{p,3}, k_{i,3}) & \text{for } t_{sw,2} < \frac{t}{t_f} \leq 1 \end{cases} \]  \hspace{1cm} (26)

IV.C.3. Saturation

As already mentioned, when the quadrotor’s velocity in any of the controlled directions is near zero, the measured Tau will approach infinite magnitude. The timing of these spikes is unpredictable, but it is still desirable to limit their magnitude. This is achieved by saturating the measured Tau signal sent to the controller at upper and lower bounds. The upper and lower bounds are defined to be equal in magnitude, at a value slightly larger than the initial reference Tau. This saturation effectively improves the robustness of the controller when operating near the singularity in Tau.

V. Flight testing and experimental results

V.A. Testing platform

The vehicle platform used for time-to-contact controller testing is the Draganflyer X4P quadrotor made by Draganfly Innovations. The off-the-shelf X4P platform has been modified significantly for research conducted on UAVs at the McGill University’s Aerospace Mechatronics Lab, incorporating various types of sensors, an onboard computer, and communication hardware. The vehicle weighs approximately 2.5kg with the flight battery and has an arm length (vehicle center to propeller center) of 33cm. The vehicle has an onboard computer running Robot Operating System (ROS); however, the majority of processing for the “auto-pilot” and the time-to-contact controllers is done on a desktop workstation running ROS, which communicates with the vehicle wirelessly.

Figure 10: X4P quadrotor used for flight testing  \hspace{1cm} Figure 11: MCPTAM camera configuration
Two different state estimation systems have been used for time-to-contact control experiments on the X4P platform. Initial tests in the Aerospace Mechatronics Lab used the Vicon motion capture system for vehicle pose estimation. Our Vicon system is comprised of six cameras mounted around the lab that monitor the positions of markers attached to the vehicle. It provides very accurate localization and Vicon measurements are typically taken as ground truth in UAV research. An Extended Kalman Filter (EKF) is used to fuse data from the Vicon system and the onboard inertial measurement unit (IMU) allowing flight controller updates at a rate of 50 Hz.

A more extensive testing campaign was carried out in a larger indoor venue at the Canadian Space Agency (CSA), where maneuvers from a wider range of heights as well as transverse landings could be experimented with. In these tests, the X4P vehicle was equipped with four wide-angle cameras and a vision-based simultaneous tracking and mapping solution developed for a cluster of rigidly-mounted cameras (MCPTAM\textsuperscript{11}) was deployed to compute the pose estimates of the vehicle. A diagram of our camera configuration for testing at CSA is given in figure 11. As with Vicon, fusion of MCPTAM pose with onboard IMU was implemented with an EKF. The hybrid-nonlinear error definition was used for the Tau controllers in all flight tests reported in this paper.

V.B. Selected experimental results

Figures 12 and 13 show logged data from two vertical Tau landings, and figures 14 and 15 show two transverse Tau landings, all carried out at CSA. The analytical functions for Tau, distance, velocity, and acceleration, as derived from the intrinsic Tau guidance equation in section III, are plotted with these data for comparison.

![Figure 12: Data from vertical Tau landing: $z_0 = 2m$, $t_f = 10s$, MCPTAM](image1)

![Figure 13: Data from vertical Tau landing: $z_0 = 4m$, $t_f = 10s$, MCPTAM](image2)

V.C. Analysis of data and discussion

V.C.1. Comments on observed behavior

There are several observable characteristics of the time-to-contact controlled landings given in figures 12-15 that are worth noting.

Oscillations in the measured Tau data are common at the beginning of maneuvers due to changes in the sign of the gap velocity when the vehicle is starting off with near zero velocity. As the velocity measurements fluctuate between small positive and negative values, the measured Tau switches between the negative and
A recurring characteristic of the experimental data, visible in the plots of measured Tau in figures 13–15, is the group of spikes in Tau close to the end of the maneuver. This occurs as the Tau controller is approaching its singularity. As the gap velocity approaches zero, when the vehicle is decelerating to rest at the end of the maneuver, the controller becomes more unstable.

The transverse Tau landing data show that the performance of the two Tau controllers ($\tau_z$ and $\tau_x$) are quite similar. Both controllers are most stable in the middle range of the maneuver, when the velocity has the positive saturation limits. These switches in sign do not change the sign of the error, and therefore the control output is in the same direction regardless of the initial velocities. However, the magnitude of the error does become larger as the measured Tau becomes positive. Practically, this is the desired behavior of the controller, although the ideal controller response would be similar regardless of whether the initial gap closure rate is $+0.001 \text{ m/s}$ or $-0.001 \text{ m/s}$.
highest magnitude. The two controllers exhibit some overshoot of the destination, and subsequently have the tendency to overcorrect for this overshoot. The overshoot of the destination is more apparent in maneuvers with a higher rate of gap closure (longer distances or shorter times), for both the $\tau_z$ and $\tau_x$ controllers. For transverse Tau landings, the $\tau_z$ controller tends to exhibit more overshoot and a larger position error at $t_f$ compared to the $\tau_z$ controller. In these maneuvers, the $x$ gap is closed earlier than the $z$ gap to produce a vertical final approach to the destination. The $\tau_x$ controller is thus hovering near zero velocity for more time, where Tau control is unstable.

V.C.2. State estimation methods

It is informative to consider the effect of the two sensing modalities used to measure the pose of the vehicle – Vicon vs. MCPTAM – and consequently the quality of the state estimation, on the performance of Tau control. Figure 16 displays the results for a vertical landing from a relatively low height. As expected, a slight degradation in controller performance can be seen between the Vicon and MCPTAM test data, with all other parameters equivalent. This comparison shows that time-to-contact control is sensitive to noise and imprecision in state estimation data, which can lead to higher variability in controller performance between maneuvers.

![Figure 16: $z_0 = 1.5m$, Vicon and MCPTAM](image)

V.C.3. Comparison to conventional landing controller

Figure 17 compares quadrotor landings controlled by time-to-contact and by velocity. The velocity-control landing controller tries to achieve a constant descent velocity until the end of the maneuver where the velocity decreases to zero. This produces a linear plot for distance vs. time, as opposed to the sigmoid plot for the time-to-contact controller. The velocity-controlled landing accelerates quickly to the desired velocity, and then decelerates quickly to rest at the end of the maneuver. The time-to-contact controlled landing, on the other hand, produces a bell-shaped velocity profile allowing for more gradual acceleration and deceleration, with a higher maximum velocity in the middle of the landing.

One of the major benefits of Tau control is the ability to arrive at a destination in a predefined time. Velocity control, or position control, could achieve this indirectly, but it would be much more difficult to account for disturbances. Tau control is thus appropriate for maneuvers where high temporal accuracy is needed. One example of this is the interception of a moving target. Another possible application is for coordinating maneuvers between multiple quadrotors.

V.C.4. Quantitative analysis of data

A summary of Tau controller performance over a series of flight tests is given in table 3. All the tests in this data set were carried out under the same Tau control scheme with equivalent control gains. The set of data is collected from vertical Tau landings with $z_0$ between 1m and 4m, and transverse Tau landings with $z_0$ and $x_0$ each between 1m and 4m. The data analysis corroborates what is visible in the gap distance plots of figures 12, 16 and described in section V.C.1. The vehicle tends to overshoot the spatial target near the end of the maneuver by an average of 5.81% in the vertical direction and 7.57% in the horizontal direction. This overshoot at low velocity results in a spike in the magnitude of measured Tau, causing a corrective action in the Tau controller that results in an average position error at $t_f$ of 8.03% in the vertical direction and 11.45% in the horizontal direction.
Table 3: Summary of controller performance from testing

<table>
<thead>
<tr>
<th>Vertical Tau landings</th>
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<td>Average overshoot (% of $z_0$)</td>
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<td>Average position error at $t_f$ (% of $z_0$)</td>
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<th>Transverse Tau landings</th>
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<td>Average overshoot - $z$ (% of $z_0$)</td>
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<td>Average position error at $t_f$ - $z$ (% of $z_0$)</td>
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VI. Conclusion

The conclusion section will summarize the findings in sections III, IV, and V. It will discuss the general applications of the angular gap closure, the control strategies used, and a summary of the results and performance on the physical vehicle. This section will discuss the pros and cons of time-to-contact control in general, and its potential applications. It will conclude with an overview of how the research will be continued in the future.

VI.A. Main findings

- The contributions of this research
- Conclusions about time-to-contact control

VI.B. Future work

- Vision-based navigation systems
- Improvement in reaching accuracy
- Coordinated timing control of multiple vehicles
- etc...

Acknowledgments

References


American Institute of Aeronautics and Astronautics


